

# Rossmoyne Senior High School

WA Exams Practice Paper A, 2015

Question/Answer Booklet

## MATHEMATICS SPECIALIST UNITS 1 AND 2 Section Two: Calculator-assumed

If required by your examination administrator, please  
place your student identification label in this box

Student Number: In figures

--	--	--	--	--	--	--	--

In words

---

Your name

---

### Time allowed for this section

Reading time before commencing work: ten minutes

Working time for this section: one hundred minutes

### Materials required/recommended for this section

#### *To be provided by the supervisor*

This Question/Answer Booklet

Formula Sheet (retained from Section One)

#### *To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,  
and up to three calculators approved for use in the WACE examinations

### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
<b>Total</b>				150	100

## Instructions to candidates

- The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer Booklet.
- You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
- Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
  - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
  - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
- Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you **do not use pencil**, except in diagrams.
- The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

**Section Two: Calculator-assumed**

**(98 Marks)**

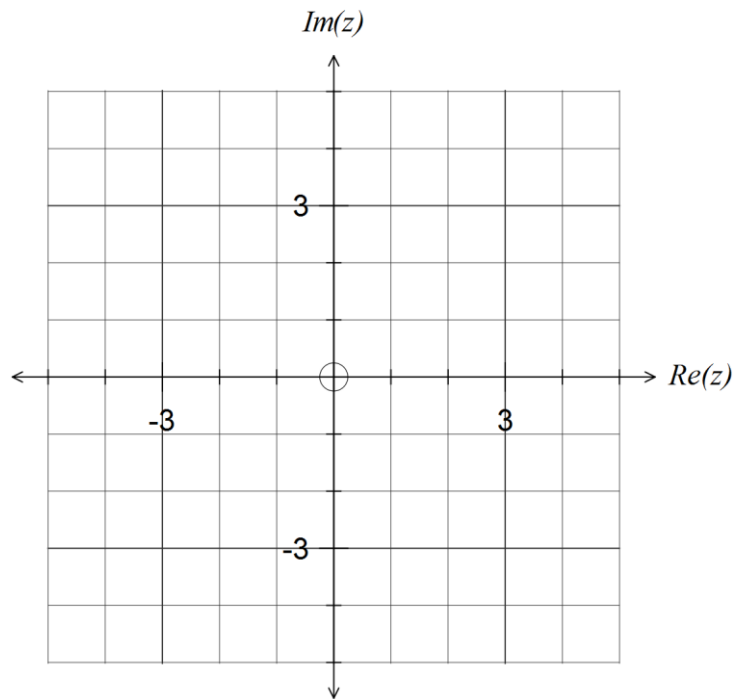
This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

**Question 8**

**(5 marks)**

- (a) Plot and label the complex number  $z = -1 - 2i$  on the Argand diagram below. (1 mark)



- (b) On the same diagram plot and label the following complex numbers: (4 marks)

- (i)  $z_1 = iz$
- (ii)  $z_2 = \bar{z}$
- (iii)  $z_3 = z^2$
- (iv)  $z_4 = z \cdot \bar{z}$

**Question 9****(6 marks)**

A boat is motoring at 18 km/h on a bearing of  $245^\circ$ . To an observer on board the boat, the wind appears to be blowing at 28 km/h from due west.

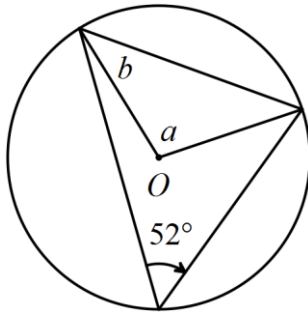
If the boat turned through  $180^\circ$  and increased its speed to 20 km/h, find the new apparent wind speed and direction.

**Question 10**

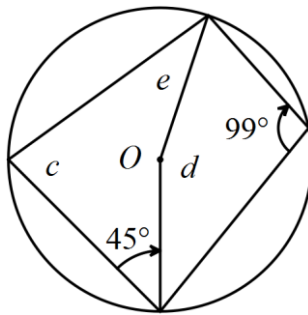
**(8 marks)**

(a) In the following diagrams,  $O$  is the centre of the circle shown.

(i) Determine the values of  $a$  and  $b$ . (2 marks)

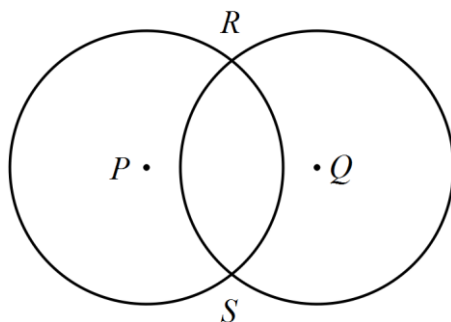


(ii) Determine the values of  $c$ ,  $d$  and  $e$ . (3 marks)



(b) Two circles, both with radius 9 cm, and centres  $P$  and  $Q$  that are 12 cm apart, intersect at  $R$  and  $S$  as shown. Determine the exact area of the quadrilateral  $PRQS$ .

**(3 marks)**



## Question 11

(7 marks)

(a) Let matrix  $A = \begin{bmatrix} 2 & -2 \\ 7 & -6 \end{bmatrix}$ .

(i) Determine  $A^{-1}$ . (1 mark)

(ii) Express the equations  $7a - 6b = 23$  and  $2a - 2b = 7$  as a system of matrices. (1 mark)

(iii) Show use of your answer from (i) to solve the matrix equation in (ii). (2 marks)

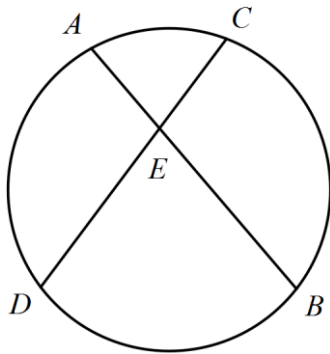
(b) Solve the equation  $\begin{bmatrix} 3 & 0 \\ 5 & 0 \end{bmatrix} B = B + \begin{bmatrix} 2 & 4 \\ 6 & 7 \end{bmatrix}$  for the  $2 \times 2$  matrix  $B$ . (3 marks)

Question 12

(7 marks)

- (a) In triangles  $ABC$  and  $DEF$ ,  $AC \cong DF$  and  $\angle A \cong \angle D$ . Is the additional fact that  $BC \cong EF$  enough to prove that triangle  $ABC$  is congruent with triangle  $DEF$ ? Justify your answer. (2 marks)

- (b) In the circle shown below, not to scale,  $AB$  and  $CD$  are chords that intersect at  $E$ . If  $AE = 4$  cm,  $BE = 8$  cm and  $CE = 6$  cm, determine the length of  $DE$ . Justify your answer. (3 marks)



- (c) Consider the true statement 'if a quadrilateral is a square then all four sides of the quadrilateral are the same length'. Write the converse of this statement and explain whether or not the converse is also true. (2 marks)

**Question 13****(8 marks)**

- (a) Determine the sum of all the numbers contained in the row(s) of Pascals triangle in which the number 10 appears. (3 marks)
- (b) Nine students applied for four temporary positions working for a cleaning company in their holidays.
- (i) How many different selections of students could the cleaning company make to fill the four positions? (1 mark)
- (ii) If six of the nine applicants were male, and the company wanted to employ an equal number of males and females, in how many ways could they do this? (2 marks)
- (iii) Determine the number of ways that the application forms can be sorted into order if the six male applications must be kept together. (2 marks)



**Question 14****(7 marks)**

- (a) A triangle with vertices at  $A(1, 1)$ ,  $B(3, 1)$  and  $C(3, 4)$  is reflected in the  $x$ -axis and then rotated  $90^\circ$  anticlockwise about the origin.
- (i) Find the matrix  $T$  that will combine these two transformations in the order given. (3 marks)

- (ii) Find the coordinates of  $C$  after transformation by  $T$ . (1 mark)

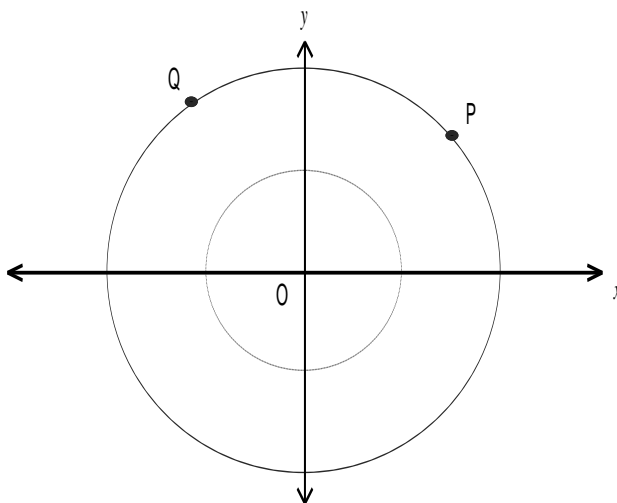
- (b) Another transformation matrix is given by  $R = \begin{bmatrix} -0.6 & 0 \\ -1.2 & -0.6 \end{bmatrix}$ .

Determine the area of triangle  $ABC$  after transformation by  $T$  and then by  $R$ . (3 marks)

## Question 15

(6 marks)

The points P and Q lie on a circle of radius  $r$  and have polar coordinates  $(r, \theta)$  and  $(r, \phi)$  respectively, where  $0 < \theta < \phi < 360^\circ$ .



- (a) Express both of the vectors  $\overrightarrow{OP}$  and  $\overrightarrow{OQ}$  in the form  $a\mathbf{i} + b\mathbf{j}$ . (2 marks)
- (b) Use your answers from (a) to show that  $\overrightarrow{OP} \cdot \overrightarrow{OQ} = r^2(\cos\theta \cdot \cos\phi + \sin\theta \cdot \sin\phi)$ . (1 mark)
- (c) Use the diagram above to state the size of  $\angle POQ$ . (1 mark)
- (d) Use the definition  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| \cdot |\mathbf{b}| \cos\theta$  and the result in (b) to show  $\cos(\phi - \theta) = \cos\phi \cdot \cos\theta + \sin\phi \cdot \sin\theta$ . (2 marks)

**Question 16****(6 marks)**

In a new office building, the manager's office is to have one desk, three filing cabinets, one fax, one telephone and four chairs.

The supervisors are to have one desk, two filing cabinets, no fax, one telephone and two chairs.

The clerks are to have one desk, one filing cabinet, no fax, one telephone and one chair.

(a) Express this information in matrix  $M$ . (1 mark)

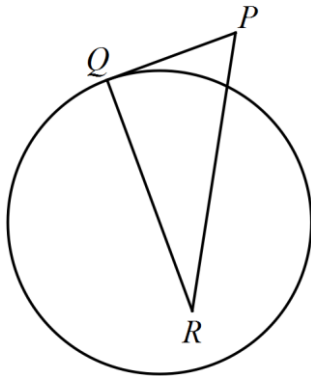
(b) In the new building there are to be 6 managers, 12 supervisors and 40 clerks. Express this information in matrix  $N$  and then show use of a matrix operation to determine the total number of desks, filing cabinets, fax machines, telephones and chairs needed. (3 marks)

(c) The cost of a desk, a filing cabinet, a fax, a telephone and a chair are \$250, \$190, \$125, \$85 and \$160 respectively. Express these costs in matrix  $C$  and then show use of a matrix operation to determine the total cost of all new furniture and office machines. (2 marks)

Question 17

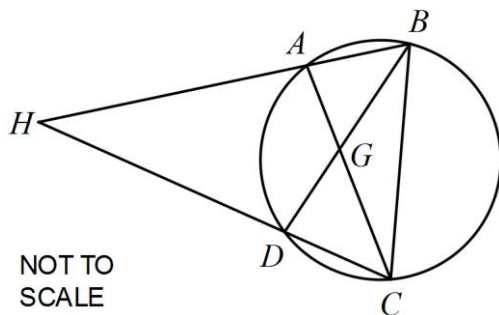
(9 marks)

- (a) The circle in the diagram has a diameter of 20 cm and  $PQ$  is a tangent to the circle at  $Q$ . If  $PQ = 7.5$  cm,  $QR = 18$  cm and  $PR = 19.5$  cm, prove that  $R$  lies on the diameter of the circle. (3 marks)



- (b) The points  $A, B, C$  and  $D$  lie on a circle of radius  $r$ . The lines  $AC$  and  $BD$  intersect at  $G$ . The lines  $BA$  and  $CD$  are produced to meet at  $H$ .  $HAGD$  is a cyclic quadrilateral.

- (i) Determine, with reasons, the size of  $\angle BAC$ . (4 marks)



- (ii) Determine, with reasons, the length  $BC$  in terms of  $r$ . (2 marks)

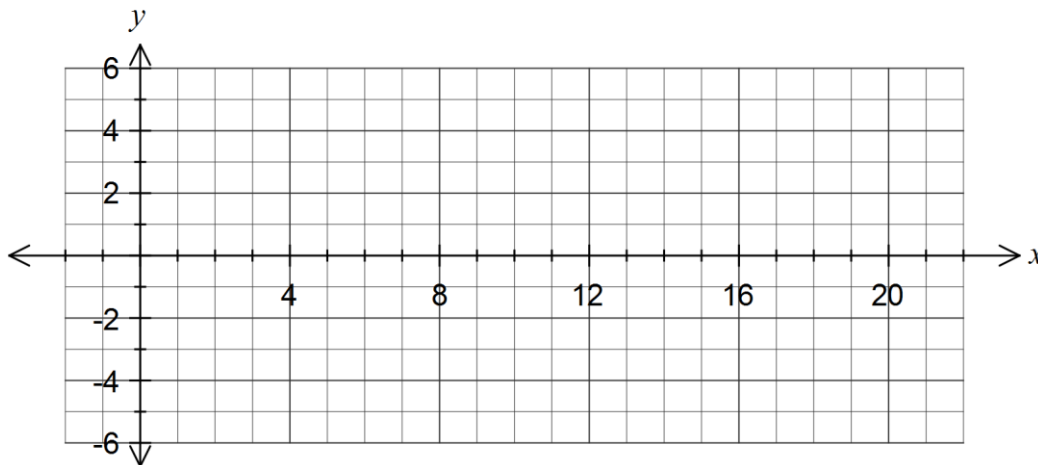
**Question 18**

**(9 marks)**

The motion of a small body moving along a straight track was recorded by a video camera for 20 seconds. An analysis of the motion showed that the distance,  $x$  cm, of the body from a fixed point  $O$  on its path  $t$  seconds after recording began was given by  $x(t) = 3\cos\frac{\pi t}{4} - 4\sin\frac{\pi t}{4}$ .

- (a) The distance can also be given by  $x(t) = a\sin\left(\frac{\pi t}{4} + b\right)$ , where  $a$  and  $b$  are real constants. Determine the values of  $a$  and  $b$ . (2 marks)

- (b) Graph  $y = x(t)$  on the axes below for  $0 \leq t \leq 20$ . (3 marks)



- (c) State the period and amplitude of the graph of  $y = x(t)$ . (2 marks)

- (e) Determine the percentage of the first 20 seconds that the body was at least four cm away from the point  $O$ . (2 marks)

## Question 19

(12 marks)

(a) Consider the expression  $m^2 + 7$ .

(i) Write down the values of  $m^2 + 7$  for  $m=1, 3, 5, 7$  and  $9$ . (1 mark)

(ii) Use your values from (a) to state the largest integer,  $p$ , that  $m^2 + 7$  is always divisible by, when  $m$  is a positive odd integer. (1 mark)

(iii) Prove that  $m^2 + 7$  is always divisible by  $p$  when  $m$  is a positive odd integer. (4 marks)

(b) Let  $A = 2^{n+2} + 3^{2n+1}$ ,  $n \in \mathbb{N}$ .

(i) Show that  $A = 7 \times 25325$  when  $n = 5$ . (1 mark)

(ii) Prove by induction that  $A$  is divisible by 7. (5 marks)

**Question 20****(8 marks)**

A helicopter, with a maximum speed through still air of 240 km/h, leaves its base at A to fly to a destination at B.

The position vector of B relative to A is  $(155\mathbf{i} + 95\mathbf{j})$  km, and a steady wind of velocity  $(-17\mathbf{i} - 22\mathbf{j})$  km/h is blowing over the area.

- (a) Find the velocity vector the helicopter pilot should set in order to fly directly from A to B in the shortest time. (6 marks)

- (b) What is the shortest journey time, to the nearest minute? (2 marks)



**Additional working space**

Question number: \_\_\_\_\_

**Additional working space**

Question number: \_\_\_\_\_

**Additional working space**

Question number: \_\_\_\_\_

This examination paper may be freely copied, or communicated on an intranet, for non-commercial purposes within educational institutes that have purchased the paper from WA Examination Papers provided that WA Examination Papers is acknowledged as the copyright owner. Teachers within Rossmoyne Senior High School may change the paper provided that WA Examination Paper's moral rights are not infringed.

Copying or communication for any other purposes can only be done within the terms of the Copyright Act or with prior written permission of WA Examination papers.

*Published by WA Examination Papers  
PO Box 445 Claremont WA 6910*